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Value-at-Risk: empirical evolution and impact on informativeness

Sofia Jerónimo Abrunhosa

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Sofia Jerónimo Abrunhosa

under the orientation of
Ricardo Cunha, PhD

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Value-at-Risk: evolução empírica e impacto na capacidade informativa

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Sofia Jerónimo Abrunhosa

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Abstract

Value-at-Risk (VaR), defined as the maximum expected loss for a certain portfolio at a target timeframe, given a certain confidence level, prevailed as the keystone indicator for market risk in the banking industry. Considering it is a generally accepted indicator among stakeholders, it is crucial to understand whether it provides an adequate level of informativeness.

Since the late nineties, it started being integrated in the quarterly and annual reports of banks, being nowadays a mandatory regulatory disclosure. This stated, the relationship between the trading VaR and the subsequent variation in trading revenues of a sample of large banks was investigated.

The results suggest that VaR has predictive power over succeeding volatility in trading revenues, presenting satisfactory informativeness for stakeholders. This confirms the investigation with no precedent performed by Jorion (2002), which served as starting point for the current investigation. Moreover, the VaR predictive power increased since the previous study, which provides evidence that the industry accommodated the many empirical evolutions reached in the VaR calculation domain over the past years.

Keywords: Value-at-Risk, informativeness, empirical evolution, market risk, reporting, Basel Committee

Resumo

O *Value-at-Risk* (VaR), definido como a perda máxima esperada para um certo portefólio, para um determinado espectro temporal e nível de confiança, prevaleceu enquanto pilar para a mensuração de risco de mercado no setor financeiro. Tendo em conta que se trata de um indicador globalmente aceite entre as partes interessadas, é fulcral entender a adequação da sua capacidade informativa.

Desde o final dos anos 90, o VaR passou a ser integrado nos reportes trimestrais e anuais publicados pelos bancos, sendo atualmente matéria de divulgação regulatória obrigatória. Neste sentido, foi investigada a relação entre o VaR da carteira de negociação dos bancos e as variações subsequentes ao nível da receita proveniente da negociação.

Os resultados sugerem que o VaR tem capacidade explicativa sob a volatilidade posteriormente experienciada nas receitas de negociação, apresentando uma capacidade informativa satisfatória para as partes interessadas.

Tal confirma a investigação sem precedentes de Jorion (2002), que serviu de base à corrente investigação. Acrescente-se ainda que se registou um aumento da capacidade explicativa do VaR desde o estudo anterior, emergindo evidências de que o setor acomodou as inúmeras evoluções empíricas que se foram registando ao nível do cálculo do VaR.

Palavras-chave: ***Value-at-Risk*, capacidade informativa, evolução empírica, risco de mercado, reporte, *Basel Committee***

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List of Abbreviations

ARCH: Autoregressive Conditional Heteroscedasticity models

BCBS: Basel Committee on Banking Supervision

BIS: Bank of International Settlements

CAViaR: Conditional Autoregressive Quantile Specification

ECB: European Central Bank

ES: Expected Shortfall

EVT: Extreme Value Theory

FASB: Financial Accounting Standards Board

FE: Fixed Effects

GARCH: Generalized Autoregressive Conditional Heteroscedasticity models

IASB: International Accounting Standards Board

IRB: Internal Rating-Based

i.i.d.: independent and identically distributed

OLS: Ordinary Least Squares

US: United States of America

VaR: Value-at-Risk

Introduction

The financial disasters experienced in the early nineties, followed by the subprime crisis in 2008, with systemic effects in the banking industry, flagged the imminent urge for the implementation of steady risk management tools and controls.

This stated, during the past years, there was a concerted effort on the banking, academic and particularly regulatory spheres towards implementing robust risk management mechanisms.

One of the main risk sources that affect the banking activity is market risk, associated with the impact of fluctuations in the financial markets in the banking book value (see Basel Committee, 1996). Value-at-Risk, introduced by Guldimann (1980s) and defined as the maximum expected loss for a certain portfolio at a target timeframe, given a certain confidence level, prevailed as the keystone indicator for market risk, being nowadays a mandatory regulatory disclosure imposed by the Basel Committee for the purpose of capital requirements calculation.

Its conceptual simplicity, aggregating in a single number the market risk of a portfolio, is in the genesis of its popularity.

However, if not properly computed, it might lead to hazardous misspecifications of the level of risk carried by an institution. An extensive effort has been performed through the past years to cover the flaws of the existing calculation methodologies and bring new ones into discussion (from RiskMetrics, 1996 parametric approach to Bali, 2007 extreme value theory approach). Moreover, ex-post assessment techniques, also known as backtesting, have been developed and even alternative measures have been considered (see Laurent, 2016 and Basel Committee, 2019).

Increasing VaR accuracy is an ongoing challenge and the current work serves exactly the purpose of understanding the evolution in terms of VaR accuracy during the past few years, through the successive assimilation of academic methodologies.

This dissertation investigates the explanatory power of the publicly disclosed VaR by some of the largest banks worldwide in the volatility of the subsequently reported trading revenues. It departs from the study developed by Jorion (2002) and updates it with some methodological variations.

The aim is to understand if, throughout the years, the financial sector absorbed the academic evolutions in the VaR calculation domain and if those evolutions reflected in an increase of informativeness for stakeholders, particularly analysts and investors.

It is fundamental for stakeholders to have access to an accurate market risk assessment, providing them with a continuous awareness of their own portfolio volatility profile and understand whether financial institutions are maximizing the efficiency of regulatory reporting and, consequently, maximizing the efficiency of capital allocation.

Chapter 1 provides a journey across the roots of VaR. Chapter 2 explores the VaR concept, the existing empirical approaches and how they compare with each other. Chapter 3 focuses on the backtesting concept and available techniques. Chapter 4 develops on the criticisms and alternatives to VaR. Chapter 5 is the core chapter, presenting the studies that preceded the current one, the methodology, the data and descriptive statistics and finally the results. The conclusion dedicates to summarizing the main topics approached, the contributions and limitations of the study.

Chapter 1
Foundations of VaR

1.1. Defining Market Risk

Among financial institutions, one of the core concerns is the risk carried in their own portfolios, the so-called market risk, which refers to changes in an investment value due to factors that generally affect financial markets.

There are four main sub risks that fit this category: the interest rate risk, associated with the impact of unexpected interest rate changes in the value of fixed income financial instruments; exchange rate risk, which refers to changes in the value of investments related to changes in the relative value of two currencies; equity risk, due to price changes in stocks; and commodity risk, associated with changes in the prices of commodities (see Basel Committee, 1996).

It is important to stress that, although risk is commonly linked with the probability of experiencing unexpected losses, it shall be viewed as the probability of unexpected outcomes, both positive and negative (Jorion, 2006). There are several past examples of significant losses associated with the neglection of extraordinary traders' performance, which should but have not raised a red flag in the respective institutions' risk management.

Market risk management and assessment plays a central role among banks, providing them a continuous awareness of their profit and losses volatility profile, and allowing them to align this profile with the institutional risk appetite policies. A rigorous and thoughtful assessment is also fundamental to maximize the efficiency of regulatory reporting and, consequently, maximize the efficiency of capital allocation (see Jorion, 2006).

1.2. The Roots of VaR

For several years, the commonly used measure to quantify portfolio risk was the standard deviation. However, the financial disasters during the early 1990's flagged an imminent urge to develop more accurate risk measures.

Exemplifying, the longstanding UK bank Barings PLC bankrupted due to losses of a single trader in derivatives. A similar situation happened with the Japanese 12th largest bank, Daiwa, but they had sufficient robustness to withstand. There is also the local government fund Orange Country incident, in the US, overexposed to reverse repos and simultaneously poorly reporting positions at cost instead of market value.

Cases like these led to billions of dollars insolvency costs and losses for both institutions and investors during the early nineties, which could have probably been avoided if portfolio VaR was publicly available, creating early awareness among investors.

Responses to the clear insufficiency of risk management policies enforcements started to be subject of debate in the private sector sphere. Value-at-Risk (VaR) models emerged in that context and have prevailed until nowadays as a keystone of market risk management in the financial sector.

Till Guldimann, the head of global research at J.P. Morgan in the late 1980's, is recognized as the father of the concept. At the time, the corporate risk management had to figure whether a full hedge was reached through investing in long maturity bonds, keeping earnings stable but market values volatile, or investing in cash, keeping the market value constant.

The conclusion was that risks associated with value volatility prevailed over risks associated with earnings, laying the foundations for the VaR concept, which was then documented for the first time in the G-30[1] report, published in 1993.

J.P. Morgan's early efforts became a material risk management tool with the issuance of RiskMetrics, which was the foundation to the following research on the risk management domain and implementations among financial institutions.

1.3. Regulatory Implementations

The transposition to the regulatory sphere happened under the January 1996 Amendment to the Capital Accord to incorporate market risk. Arising from the Basel Committee on Banking Supervision intention to refine their framework towards the incorporation of banks' exposures to the different classes of securities, it allowed banks to base their regulatory capital requirements for market risk on internal VaR calculations. Naturally, they were subject to qualitative and quantitative standards (see BCBS, 1996). A simultaneous implementation, under the FASB (Financial Accounting Standards Board) and then IASB (International Accounting Standards Board), was the recognition of derivatives as on-balance-sheet items, with a similar treatment to any other financial instrument, imposing a mark-to-market valuation.

It was the Basel II Accord, in 2004, that settled the incorporation of market risk in the regulatory framework. It is anchored in three pillars: minimum regulatory risk-based capital requirements, setting charges for credit, market and operational risk; supervisory review, enhancing the role of the regulator in assuring that financial institutions operate below the minimum capital ratios; market discipline, implementing a new disclosure calendar and publishing

[1] The Group of Thirty – Consultative Group on International Economic and Monetary Affairs

requisites to increase reporting discipline and, consequently, stakeholder awareness.

The capital requirements for market risk can be calculated through the standardized approach, called the building block approach, which suggests the computation of market risk for portfolios exposed to interest rate risk, exchange rate risk, equity risk and commodity risk, which are then summed to get the full market risk. The main downside of this approach is ignoring diversification across risk categories, causing an over conservative capital charge.

On the other hand, there is the internal rating-based approach, in which banks are allowed to use own models, as the regulator recognized that banks have developed much more sophisticated methodologies internally. Naturally, they have to satisfy qualitative and quantitative standards.

Independently of the chosen method, the VaR shall follow these assumptions: 10 trading days horizon, 99% confidence level, historical observation period of 1 year or more (see BIS, 2004).

Later, in response to the 2007 to 2009 great financial crisis, catalyzed essentially by excessive leverage and insufficient liquidity buffers across the banking industry, combined with poor risk management and incentive schemes, a package of additional principles and documentation was released to strengthen Basel II regulatory framework.

It was only in 2010 that Basel III standards were released, paving the path for a complete restructuring on the regulatory framework, by starting with incorporating higher capital standards and formulating improved liquidity risk measurement standards. This reform is a phased in process, working in the strengthening of each of the Basel pillars, which is expected to be complete by the end of 2019 (see BIS, 2004).

In what concerns the standards for market risk minimum capital requirements, the revision was launched in 2016 and is expected to be fully implemented by the end of 2019. Its core contributions are a product of the flaws identified in the great financial crisis. It contemplates a revision of the trading book and banking book concept, a revision of both the standardized and the internal models' approach, a shift from value-at-risk (VaR) to expected shortfall (ES) and an introduction of market risk associated with illiquidity.

Focusing on the shift from VaR, as defined in Basel II, to ES, it basically means going from a VaR estimation of maximum expected loss to a Conditional VaR estimation of expected losses beyond the VaR threshold. This stated, ES is a derivation of VaR, it follows the same assumptions but takes a further step by producing a measure of the scale of losses beyond VaR. The intention is to produce a more prudent measure, allowing for a sturdier tail risk assimilation and capital adequacy to react to stress periods.

There is strong academic debate on the merits of these two, as explored by Laurent (2016). On the one hand, the ES approach is sub additive, allowing for positive externalities from diversification, however, it is claimed to be strongly dependent on scarce extreme events, triggering lack of robustness issues.

The committee attenuates the potential over optimism of the above diversification effect by implementing limits.

In terms of the calculation criterion for ES, a Gaussian distribution is assumed in the standardized approach, a confidence level of 97.5% and a time horizon dependent on the liquidity horizon, by risk factor and not by financial product type, to mitigate the risk of an unexpected loss of liquidity across asset markets (see BIS, 2019).

Naturally, the consistency of this new implementation is a long way from being certain, both from an intellectual and practical point of view, as Laurent (2016) claims and further develops.

Chapter 2
Concept and Evolution of Empirical Approaches

Even though VaR's elementary purpose was risk assessment, and, consequentially, capital requirement definition, providing protection against catastrophic market events in an increasingly globalized financial system, it is also broadly used as a management tool. For example, when the management decides on the traders' capital allocation and trading limits and even at a later stage, as a guide to portfolio choice (for example, an increasing VaR might trigger the closing of a position). It is even becoming increasingly relevant in a non-financial environment, applied to variables other than securities returns.

Conceptually, VaR is a statistical measure of maximum expected loss for a certain portfolio, at a target timeframe, given a certain confidence level.

Its conceptual simplicity, aggregating in a single number the market risk of a portfolio, is in the genesis of its popularity.

Furthermore, it appears as a coherent risk measure, by satisfying the following desirable properties proposed by Artzner et al. (1999) for capital adequacy matters:

- Monotonicity: if portfolio X1 has consistently lower returns than portfolio X2, then X1 has higher risk than X2.
  $X1 \leq X2$, then $\varrho(X1) \geq \varrho(X2)$
- Subadditivity: a portfolio's risk cannot be higher than the sum of the individual securities or smaller portfolios risks.

$\varrho(X1 + X2) \leq \varrho(X1) + \varrho(X2)$

- Translation invariance: adding k cash to a portfolio shall reduce its risk by k.

  $\varrho(X + k) = \varrho(X) - k$

- Homogeneity: increasing the size of a portfolio by α, increases the risk proportionally.

  $\varrho(\alpha X) = \alpha\varrho(X)$

VaR estimation methodologies fit in three main categories of approaches, Nonparametric, Parametric and Semiparametric, as defined by Manganelli and Engle (2001). The Monte Carlo simulation method is often fitted in the parametric category but, considering its peculiarities, may be considered as a fourth category.

2.1. Existing Approaches

The choice of the stochastic process for prices is fundamental because if it is not realistic, the estimate of VaR will not be representative of the actual potential losses. Several approaches have been presented throughout time.

2.1.1. Nonparametric Approaches

The most general methodology to calculate VaR is the so-called nonparametric VaR. It is free from any assumption on the shape of returns' distribution, using the past returns to predict future returns. The Historical Simulation approach falls under this category, focusing on the collection of returns on a time frame of observation (usually between one and three years), organizing them in ascending order to be able to determine the desired left quantile of returns (according to the defined confidence level) and directly obtain the VaR number.

By making no assumption at all regarding returns' distribution, it is making a strong implicit assumption that the distribution of portfolio returns is i.i.d. and does not change within the timeframe, generating some inconsistencies (see ECB, 2001).

One of them relates to the choice of the time frame, which shall have enough length to allow a statistically significant inference but simultaneously shall capture only the current volatility cluster, which is by itself hard to identify.

Moreover, if the VaR reports to a moment when the market is changing from low-to-high or high-to-low volatility periods, the estimate will be biased until time enough has passed so that the previous volatility frame leaves the observations window.

But probably the most determinant inconsistency is the existence of groundless estimation jumps associated with the discreteness of extreme returns. This means that following a day with a large return, the estimate of VaR will jump in the opposite direction and a reversed effect will happen when the large return observation drops out of the observation time frame (for example, one year later if we consider a 1-year historical VaR).

To alleviate these identified issues, a hybrid approach was presented by Boudoukh, Richardson and Whitelaw (1998), which attributes exponentially decreasing weights to past returns within the selected time frame, attributing more weight to recent returns than to older ones, allowing for a quicker response to changes in the market panorama.

These nonparametric or quasi nonparametric methods present a simplistic, free from aprioristic assumptions on distributional shape, and easy to calculate approach comparing to parametric methods. However, there is some compromising in terms of results accuracy that shall be duly weighted.

2.1.2. Monte Carlo Simulation

The Monte Carlo methodology, first formulated by Stanislaw Ulam in the atomic science context, and then furtherly explored by Nicholas Metropolis and John Neumann, in the 1940s, was latter applied to the financial risk management domain as an alternative to the above. Its essence stems from repetitively simulating a random process for the variable under scrutiny, enough to present an extensive series of possible outcomes.

However, this method is not detached from distributional assumptions, since one must define the underlying standard joint distribution and specify a mean vector and a covariance matrix.

It has a vast potential and flexibility, allowing for the incorporation of various risk sources and complex interactions, as well as the inclusion of non-linear exposures and complex pricing profiles (for example, options) or even more complex models to characterize expected returns.

Yet, computing such specifications and complexity requires time and expensive investments in intellectual and systems development. If one does not want to compromise in accuracy, the number of simulations might easily reach astronomical figures, which is why the research towards new alternative methodologies is an ongoing concern, seeking for a more reasonable compromise between accuracy and complexity (see Jorion, 2006).

2.1.3. Parametric Approaches

Moving on to the category of parametric VaR approaches, here fit those models which imply the definition of a certain shape for returns. The earlier models assumed a standard normal distribution of returns, namely the JPMorgan RiskMetrics (1996) - which can be classified as an integrated GARCH[2], inferring

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[2] Generalized Autoregressive Conditional Heteroscedasticity models

variance through the exponential weighted moving average – and normal GARCH.

When measuring market risk, capturing the financial data volatility is one of the key concerns. The ARCH[3] models, introduced in econometrics by Engle (1982), exhibited good adherence to financial data, as they are able to differentiate conditional and unconditional variance and allow conditional variance to change over time. Additionally, GARCH models, by introducing a moving average component, allow modelling both conditional changes in volatility over time and changes in time-dependent variance.

However, because they rely on the assumption that standard residuals are i.i.d., implying normally distributed, this set of approaches tends to underestimate VaR. This because there is strong evidence that financial returns tend to deviate from that shape, by often presenting fat-tails and excess kurtosis (versus the symmetry and kurtosis of 3 of the normal distribution), which means that one might be underestimating the size of expected losses (topic addressed, for instance, by Hansen, 1994, Harvey and Siddique, 1999, Jondeau and Rockinger, 2003).

This stated, in practice, VaR results under the normal distribution may be suitable in an environment of normal and stable market conditions but may fail in accommodating exceptionally volatile periods.

Along with this potential misspecification source, it is also important to verify the empirical robustness of the variance specifications applied. The RiskMetrics Integrated GARCH model, for example, infers variance through the exponentially weighted moving average model, which accounts for the volatility-clustering phenomenon but fails to capture the asymmetry in stock market volatility and leverage effects (Pagan and Schwert, 1990).

[3] Autoregressive Conditional Heteroscedasticity models

These models have the great advantage of providing a complete characterization of returns' distribution at a relatively low computational cost and there is room for improvement on its potential misspecifications by adjusting the distributional assumptions.

Considering the identified drawbacks, many variations within the parametric models have emerged. Some of them are evolutions of the standard GARCH model, incorporating asymmetry and leverage effects. See, for instance, AGARCH[4] of Engle (1990), EGARCH[5] of Nelson (1991), GJR-GARCH[6] of Glosten, Jagannathan and Runkle (1993), VGARCH[7] and NAGARCH[8] of Engle and Ng (1993), APGARCH[9] of Ding, Granger and Engle (1993), TGARCH[10] of Zakoïan (1994), QGARCH[11] of Sentana (1995), SQR-GARCH[12] of Heston and Nandi (2000).

There are other variations in the domain of the stochastic volatility models, introduced by Taylor (1982, 1986). These models detach volatility from historical observations and attach it to an autoregressive stochastic process. While in the GARCH model the estimation of parameters can be calculated by maximum likelihood techniques, in stochastic volatility models alternative processes had to be conceived.

Another variation proposed is the realized volatility method, concept introduced by Merton (1980), further explored by Taylor and Xu (1997) and latter corroborated by Andersen et al. (2001) and refined by other subsequent authors. The basic notion is that daily volatility can be easily inferred through the addition

[4] Asymmetric GARCH model
[5] Exponential GARCH model
[6] Threshold GARCH model of Glosten
[7] Vector GARCH model
[8] Nonlinear Asymmetric GARCH model
[9] Asymmetric Power GARCH model
[10] Threshold GARCH
[11] Quadratic GARCH model
[12] Square-Root GARCH model

of the intra-daily square yields. One of the main consensuses to be reached in this model is the optimum basis frequency.

With the aim of taking into account asymmetry and excess kurtosis of financial returns, it is also possible to identify academic efforts towards the implementation of the t-student distribution for VaR calculation, which reached a good acceptance in the industry, although academically the conclusions are ambiguous. Nonetheless, the classic t-student distribution has the main setback of not capturing financial returns skewness, which is why some modified versions arise, such as the Skewness t-Student Distribution (SSD) of Hansen (1994), Exponential Generalized Beta of the Second Kind (EGB2) of McDonald and Xu (1995), Skewness t-Generalised distribution (SGT) and Skewness Error Generalised Distribution (SGED) of Theodossiou (1998; 2001).

2.1.4. Semiparametric Approaches

In that context, alternative semiparametric models, which are an arrangement between the parametric and nonparametric approach, emerged and have been further explored in recent years. The ones being approached here, for the relevance to them attributed, are the CAViaR (Conditional Autoregressive Quantile Specification) and the EVT (Extreme Value Theory).

The CAViaR proposes modeling the quantile directly instead of the whole distribution. It was introduced by Engle and Manganelli (2004) and departed from the above empirical statement that financial returns tend to present volatility clusters throughout time, which means that its distribution is autocorrelated. To formalize this specificity, an autoregressive specification was employed and the parameters were estimated by regression quantiles (introduced by Koenker and Bassett, 1978), which was considered more robust than OLS estimators when errors are fat-tailed.

Moving to the Extreme Value Theory, one of its latest advances was introduced by Bali (2007), who presents an unconditional and conditional extreme value approach to calculate VaR. The aim is to demonstrate that using the distribution of extreme financial returns provides a more accurate estimate of VaR than recalling the distribution of all returns, incorporating simultaneously a satisfactory prediction of cataclysmic market events.

The author's starting point was the unconditional extreme value approach developed by Longin (2000), McNeil and Frey (2000), and Bali (2003), incorporating the statistical theory of extremes and the corresponding tail estimation. However, the unconditional approach ignores the serial correlation and conditional heteroscedasticity of most financial data, so a conditional approach is proposed to account for systematic time-varying fluctuations in financial returns distribution.

The results indicated that specifically the conditional Box-Cox GEV [13] distribution reached an outstanding performance on capturing the extent and rate of occurrence of extreme market events, outperforming both the normal and skewed t distributions.

2.2. Comparing the Models

Although conceptually simple, if not properly estimated, VaR may lead to inaccurate definition of capital levels by financial institutions, which might turn excessively high or low, producing inefficiencies on risk coverage or capital allocation.

From this issue, several authors committed to compare the existing approaches, to identify those which represent more appropriately the actual distribution of returns. It is hard to find work documenting the comparative

[13] Conditional Box-Cox transformation applied to the generalised distribution of extreme values

performance of a wide range of VaR methodologies, as the comprehensive review and summary table presented by Abad, Benito and López (2014) illustrates. Most of the identified papers focus on comparing the extreme value theory with either the nonparametric historical simulation or variations of the parametric approach.

This derives from the fact that the EVT based methodologies keep proving to produce the best results alone and comparatively, so they are often seen as the benchmark for alternative approaches.

Still, in earlier studies it is possible to find comparisons focusing on parametric and nonparametric approaches, considering the alternative methods were not as developed yet. Within the parametric family, the assessment of alternative approaches to VaR is elaborated by Bams, D., and Wielhouwer, J. L. (2000). The authors compared four methods.

The unconditional method is the simplest of all and the one broadly applied among financial institutions. It assumes constant variance and expected value of returns over time. When using this method, the historical period used shall be short, otherwise, the obtained results are susceptible of being distant from the present situation. The second method, which considers time varying drift, applies an AR model, incorporating the existence of persistence in the levels of returns.

Another method with time varying drift and volatility, being the error term normally distributed, by applying GARCH (1,1). Finally, one with time varying drift and volatility, also making use of GARCH (1,1) but being the error term t-student distributed, to allow for a fatter left tail, meaning more probability of high losses.

Among the briefly described models, the one that proofed more accuracy for the purpose was the last one, as it combines a progressively higher weight attributed to more recent observations, which are more likely to influence the near future and, simultaneously fat left tails, which is a more realistic view on securities, that have the tendency to show more frequently high losses than high positive returns. However, this comes at the cost of having a larger standard error, meaning more uncertainty about the true value.

The conclusions reached underline the importance of establishing an adequate probability distribution, including fat tails that better reflect the behavior of extreme returns.

Moreover, being the VaR based on historical information, more weight should be given to recent observations, which better reflect the current market conditions. There are two alternatives in doing so, introducing a time varying return distribution on a large sample of historical returns or use more recent observations only. The first option is preferable, once it allows for more robust estimates.

It also became evident that the VaR should not be presented alone but preferably with the respective standard deviation, to provide insight on the underlying level of uncertainty.

Succedingly, Kuester et al. (2006) performed a broader comparison between variations of the nonparametric, parametric approaches and EVT. The basis for comparison were historical daily returns of the NASDAQ Composite Index over 30 years and the Christoffersen (1998) framework was applied to evaluate the out-of-sample forecast accuracy. The authors found that the heavy-tail GARCH under an EVT based approach demonstrated the best performance, followed by a filtered historical simulation. Once more, using distributions that account for fat-tails and skewness constitutes an improvement contrasted with the normal assumption, being the skewed t within the best performing models.

It was also possible to conclude that none of the CAViaR models performed well, which is associated with the absence of a return process estimation along with the direct quantile modelling. So, the authors suggest a multistep-ahead forecast that estimates a model for the overall return profile and a CAViaR model for the quantiles.

Moving on to Ergun and Jun (2010) work, which focuses on GARCH and ETV methodologies comparison when applied to the S&P500 Index, a flag is raised about the potential of the GARCH models. Specifically, the ARCD model (Autoregressive Conditional Density) parametric approach, which allows for time-varying conditional high-order moments, exhibited the most accurate results, when compared to other GARCH models or the EVT approach, which still presented good results.

Bali and Theodossiou (2008) and later Polanski and Stoja (2010) reinforce that the parametric method, allowing for asymmetric and leptokurtic distributions and in a mixed-distribution framework, provides accurate forecasting power, which strengthens the statements of the above paragraph.

In summary, it is possible to conclude from most literature dedicated to VaR model comparison that EVT is the common denominator of most studies, and performs the best when estimating VaR, followed by the Filtered Historical Simulation. The parametric techniques, when anchored on fat-tailed and skewed distributions and coupled with time-varying conditional high-order moments, also present great potential of VaR estimation (see Abad, Benito and López, 2014).

Chapter 3
Backtesting VaR Calculations

3.1. The Concept

Backtesting constitutes an ex-post assessment on the performance of a certain model. Sound backtesting procedures add a layer of confidence in the conceived model by showing how well they behave when applied to real historical events.

In the case of VaR backtesting, it is essential to choose a representative sample of historical data, covering a period long enough to capture potential variations on market conditions and events. It is important to avoid being biased when choosing the sample, because when formulating VaR measures, historical data is often used in the model's calibration, so the risk management shall backtest under different data sets.

Backtesting procedures intend to capture if the number of exceptions, when comparing the daily VaR to the real daily gains or losses, exceeds the chosen confidence level. A higher number of exceptions means a less accurate VaR model, which is underestimating risk. On the other hand, if the number of exceptions is too low it might mean that the institution is inefficiently allocating capital for market risk purposes (see Escanciano and Olmo, 2010 and Jorion, 2006).

Lastly, it is important to stress that backtesting is not the same as scenario analysis, as the second, instead of real historical data, relies on hypothetical data to simulate potential outcomes.

3.2. Regulatory Approach

As stated by Alan Greenspan (1996) and very suitably quoted by Jorion (2006) in the backtesting context, *disclosure of quantitative measures of market risk, such as value-at-risk, is enlightening only when accompanied by a thorough discussion of how the risk measures were calculated and how they related to actual performance.*

Particularly in a context in which VaR is the recognized cornerstone indicator of market risk and adding the fact that the Basel Committee allows financial institutions to report VaR under internal based approaches, it is fundamental to have mechanisms to validate VaR models accuracy.

The Committee's backtesting framework (see BIS, 2019) requires institutions implementing the internal based approach to compare their daily VaR number with the subsequent daily trading gains or losses. To extract the banks' VaR model performance, one shall count the number of times that VaR surpasses the following trading revenue in absolute terms (number of exceptions). The obtained coverage ratio is then compared to the goal coverage of the model to obtain its performance measure.

Materializing, for the 97.5th and 99th percentile, with at least the most recent full year of observations equally weighted, if the number of exceptions exceeds 30 or 12, respectively, the institution shall migrate back to the standardized approach. The institution shall stick to the standardized approach until it is able to demonstrate to be below the exceptions' thresholds over at least the previous full year.

But the Committee does not rely solely on this backtesting measure. In order to increase the backtesting informativeness, it defines alert levels below this threshold based on the number of exceptions and the corresponding probability of model inaccuracy.

Table 1 shows an example for 250 observations (one trading year):

Table 1 *- Basel III Backtesting Zones*

Zone	Number of Exceptions	Multiplier	Cumulative Probability[14]
Green	0	1.50	8.11%
	1	1.50	28.58%
	2	1.50	54.32%
	3	1.50	75.81%
	4	1.50	89.22%
Yellow	5	1.70	95.88%
	6	1.76	98.63%
	7	1.83	99.60%
	8	1.88	99.89%
	9	1.92	99.97%
Red	≥ 10	2.00	99.99%

In the green zone, there are no reasons to concern about the model's accuracy, with less than 4 exceptions. In the yellow zone, the number of exceptions is already a flag of a necessary increase in capital requirements for market risk by the multiplication factor. The logics behind the multiplier is being enough to restore the 99th percentile standard. Finally, in the red zone, it is assumed that there is a problem inherent to the model and the regulator starts a deeper investigation on the circumstances and requests the institution for immediate efforts towards improving the model.

The intention is to instigate the financial institutions' risk management to have robust and ongoing monitoring procedures and create steady fundamentals for

[14] The probability of obtaining a given or fewer exceptions in 250 observations when the true coverage level is 99%.

their proposed models. This is the Committee's first step in the direction of a robust verification system.

3.3. Academic Approaches

The effort to appropriately backtesting VaR models goes beyond the regulatory sphere. In the academic domain, different approaches were experimented.

Firstly, the simplest approach is the unconditional coverage, which ignores that data is time varying. This is an important issue, as it is different to have a condensed cluster of backtesting exceptions or exceptions scattered over time. When clustered, they might not mean an actual model invalidity but some specific abnormal market event.

The author of the broadly acknowledged unconditional coverage test himself, Kupiec (1995), recognizes the above weakness.

To cover this issue, conditional coverage models emerged, starting from Christoffersen (1998) Markov test, which examines the independency of VaR models. It consists on daily setting a deviation indicator of 1 or 0, depending on VaR being exceeded or not, respectively. This indicator, when treated under the appropriate formulated test statistics, allow us to understand whether exceptions are clustered or not. He later added to this formulation that if exceptions are independent from each other, the time frame between them shall be independent from the time since the last exception occurred, meaning exceptions to VaR shall not present duration dependence.

In parallel, some authors suggested the evaluation of the full distribution instead of focusing solely on a certain percentile (see Crnkovi and Drachman, 1997 and Berkowitz, 2001), however this method demands a large historical interval of data to produce significant results.

There is also the class of loss function based backtesting, introduced by Lopez (1999), which goes further than a hit function of violating or not the estimated VaR and captures the magnitude of the exceedance. In this case, the accuracy of the VaR models is determined taking into account the average losses on exceedances instead of solely the number of violations.

Following the intention of moving from binary variables to more in-depth backtesting methods, there is also the quantile regression method, treating VaR as the regressor, based on Koenker and Xiao (2002) quantile regression model, and further developed by Gaglianone et al. (2011). This methodology allows the identification of when and why a misspecification occurred.

Chapter 4
Criticisms and Alternatives to VaR

With the methodological evolution over time, as presented in Chapter 2, much of the identified VaR flaws were progressively accommodated.

However, there are some remaining VaR drawbacks that are inherent to its concept and context, and not as much related to the technicalities under its calculation.

Context wise, it is argued that the increasing complexity under VaR modeling favors larger financial institutions, considering they have larger risk management structures and resources comparing to smaller ones or even to the regulatory bodies (see, for instance, Mariathasan and Merrouche, 2014 and Begley, Purnanandam and Zheng, 2016). The possibility of internal rating-based reporting allows them for great discretion that might lead to unidentified underreporting, compromising the fairness and adequateness of capital allocation. Backtesting methodologies are used by the regulatory authorities to prevent this hazardous behavior from happening. However, considering backtesting is based on historical asset prices, there is the possibility of going undetected or of delayed detection. It is arguable if the flaw is on VaR and its inherent calculation complexity or on the backtesting techniques, but the ongoing concern with techniques accuracy and good practices tends to progressively mitigate this issue both on the regulatory and institutional spheres.

Moving to the conceptual setbacks, one of the main concerns relates to the fact that VaR smooths volatility estimates and, consequently, appears as a fragile indicator during severe crisis periods, as the 2008 crisis confirmed (as shown by Haldane and Madouros, 2012, for example).

This is in the basis for the shift from VaR to ES, proposed in the Basel III framework, as the latest allows the quantification of the scale of losses beyond VaR. Yet, this alternative has not reached unanimous support, as it is strongly dependent on scarce extreme events, triggering lack of robustness issues. Furthermore, being a sub-additive measure that innately accounts for diversification benefits, might lead to an over optimistic incorporation of diversification effects. Laurent (2016), provides a clear summary of the benefits and setbacks of this alternative and leaves a question mark on the tradeoff that can be obtained from the migration, under a financial stability perspective.

Nevertheless, neither VaR not its most popular alternative, ES, are as straightforward or as transparent as they conceptually appear. As stated by Laurent (2016), the IRB approaches are complex and limitless, which combined with the specificities in the nature of each bank's portfolios, makes model auditing and comparisons between institutions difficult.

It is arguable if this is a problem of the models themselves or an issue at the regulatory framework level, often criticized, namely by Danielsson et al. (2015), for making it difficult to implement reliable macroprudential policies.

Probably there are sufficient arguments to consider a market risk assessment less reliant on VaR, incorporating complementary measures, such as ES (to account for the scale of losses beyond VaR), diversification effects metrics, and asset classification, weighting them in a single equation. Furthermore, weighting the benefits of allowing for IRB approaches instead of a single, generally accepted in the academic field methodology, is also an important topic to put under discussion.

In summary, it is doubtless that VaR is a keystone on market risk measurement, what is under scrutiny in most literature is its legitimacy without the support of complementary measures, its regulatory framework and the accuracy of each alternative methodology for its calculation.

Chapter 5
Testing VaR Disclosures Informativeness

5.1. Past Studies

Understanding the effectiveness of VaR calculations in predicting the unexpected future potential maximum losses is fundamental to conclude about its pertinence as the key market risk measure among financial institutions.

To do so, in an assessment with no precedent, Philippe Jorion (2002) in *How Informative Are Value-at-Risk Disclosures?* centers on the relationship between VaR disclosures in financial institutions reports and the subsequent behavior of their trading returns.

Berkowitz and O'Brien (2001) published a paper addressing the daily VaR privately reported by Banks and the subsequent trading returns. However, this study acts at the estimation accuracy level but not at the level of informativeness of publicly disclosed VaR estimates to the overall stakeholders. This because daily VaR numbers are private information, exclusive to the regulators and not available to the remaining stakeholders. Naturally, the informativeness of daily VaR on subsequent trading revenues is expected to be satisfactory, but the same might not hold in quarterly or annual frequencies.

To perform the investigation, Philippe Jorion (2002) used a sample of eight US banks, considering at the time the study was performed it was not possible to find more banks with at least five years of publicly disclosed VaR, on quarterly or annual reports.

The results showed a very significant relationship between VaR forecasts published by banks and the future experienced market risk, particularly when using cross-section analysis but also over time. This means that banks which reported low VaR had a narrower downside risk and those with higher VaR experienced larger fluctuations.

Considering the obtained conclusions, VaR reports appeared to be meaningful flags for information stakeholders, as it is the case of analysists and investors, in the evaluation of risk/ return adequacy.

Apart from the usefulness of VaR reporting as a relevant information source for stakeholders, it is also a mechanism to impose discipline inside the institutions towards constant monitoring and risk management.

Nonetheless, there was still plenty of room to accuracy improvement, even within this sample of well-established financial institutions. At the time of the publishing, the author claimed there shall be a natural evolution as new and more consistent methodologies arise.

More than 15 years later, that is what the current work intends to infer, with considerably more data available and a sufficiently large time frame behind to allow the financial industry to incorporate the methodologies that have proven more accuracy.

5.2. Methodology

VaR can be defined as the maximum expected losses, being the probability of an institution incurring losses beyond that level 1 minus the confidence level defined (for instance, for the commonly used confidence level of 99%, the probability of incurring losses beyond the VaR level would be 1%).

Interpreting the above, the probability of the absolute unexpected trading revenue of the following period ($Rev_{t+1} - E(Rev_{t+1})$) being lower than the reported VaR on the current period (VaR_t) shall be 1 minus the confidence level (cl):

$$P[|Rev_{t+1} - E(Rev_{t+1})| < VaR_t] = 1 - cl$$

However, we are not able to perform this daily computation, because daily VaR and trading revenues data is proprietary information a stakeholder cannot access.

This stated, and as an alternative, as suggested by Jorion (2002), VaR will be transformed into a dispersion measure, as tests based on dispersion are more powerful than those based on the number of exceptions (analogously to what happens in backtesting, as described in 3.2.). To do so, it was assumed that trading revenues (Rev_{t+1}) are symmetrically distributed while accommodating fat tails, which, as previously referred, is a characteristic of financial returns.

Translating the above, trading revenues are assumed to have a conditional normal distribution with mean zero and variance s_t^2.

$$Rev_{t+1} \sim N(0; s_t^2),$$

being $s_t^2 = \omega_t' \sum_t \omega_t$, which represents the effect of fixed positions multiplied by the covariance matrix of market risk factors at day t closing.

This approach becomes distorted when the underlying distribution is much skewed, but this shall not be a concern, considering the current sample only includes large banks, which typically carry on their own portfolios a large range of instruments, exposed to the most diverse risk factors.

Incorporating these assumptions, means that VaR_t will be the forecasted volatility times the standard normal deviate assumed, associated to the confidence level underlying VaR calculation (the most common is a 99% confidence level, meaning a standard normal deviate of z = 2.33):

$$VaR_t = zs_t$$

The last step to transform VaR in a volatility measure is performing the square root of time adjustment[15]. Then, the estimated volatility adjusted to the period (quarter or year) will be:

$$\sigma_t = s_t\sqrt{T}, \text{ being } s_t = \frac{VaR_t}{z}$$

Applying a 99% confidence level and quarterly VaR and trading revenues reporting, the estimated volatility of trading revenues at time t is:

$$\sigma_t = \frac{VaR_t}{2{,}33}\sqrt{63}$$

The above implies that one assumes σ_t is measured without error. Extrapolating VaR to quarterly or yearly horizons might be a source of error, by assuming the variance in trading revenues is constant over time. By examining the daily VaR graphics disclosed by some of the large banks, this assumption appears reasonable, as there is some variation within the quarterly and yearly frames, but major variations occur in longer horizons.

Additionally, it is necessary to create a measure of unexpected trading revenues. This because, there is an expected component in trading revenues related to fees and interest income, which is out of the scope of VaR calculations.

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[15] In circumstances with i.i.d. returns, variances are additive throughout time, meaning volatility grows with the square root of time. For this purpose, time is measured in trading days (252 in a year, 92 in a quarter).

To do so, and once more in line with the suggested by Jorion (2002), the moving average of trading revenues over the past four quarters or the past year, depending on the availability, is subtracted to the quarterly or annual trading revenue under computation. The unexpected trading revenue can then be defined as follows:

For quarterly reported values:

$$Rev_{t+1} - E[Rev_{t+1}] = Rev_{t+1} - \frac{\sum_{i=1}^{4} Rev_{t+1-i}}{4}$$

For yearly reported values:

$$Rev_{t+1} - E[Rev_{t+1}] = Rev_{t+1} - Rev_t$$

Summarizing the above specifications, the final regression under scrutiny will be:

$$|Rev_{t+1} - E[Rev_{t+1}]| = a + b\sigma_t + \varepsilon_{t+1}$$

We are then testing if the volatility based on the average VaR reported at period t is informative about the unexpected trading revenues verified on the front period. The expectation is obtaining b>0 to confirm the relationship and a high R^2, meaning VaR captures much of the unexpected trading revenues.

5.3. Data and Descriptive Statistics

The first criterion for sample choice was confining the study to large banks, which are jointly fairly representative of the financial sector. Furthermore, large banks tend to have sturdier risk management structures and, consequently, are usually pioneers in applying the most sophisticated methodologies and the latest academic findings. This stated, the following sample of sixteen of the largest banks worldwide was used: Banco Bilbao Vizcaya Argentaria, Banco Santander

SA, Barclays PLC, BNP Paribas, Credit Agricole, Credit Suisse Group AG, HSBC Holdings PLC, Intesa Sanpaolo, Lloyds Banking Group PLC, Nordea Bank ABP, Royal Bank of Scotland Group, UniCredit SPA, Wells Fargo & Co, JPMorgan Chase & Co, Société Générale, Royal Bank of Canada.

Table 2 shows that the sample is formed by large banks that jointly have a 24.64% market share on the banking industry (measured in terms of total market cap), with individual market shares varying between 5.67% (JPMorgan) and 0.31% (Société Générale). Furthermore, it is important to emphasize that there are banks from several counties, so the study is not constrained to United States banks, as it was the case on Jorion (2002) previous study.

***Table 2** - Sample Banks Country and Market Share*

Institution	Market Cap (billion EUR)	Country	Market Share (%)
JPMorgan Chase & Co	357.07	United States	5.67%
Nordea Bank ABP	316.63	Finland	5.03%
Wells Fargo & Co	212.43	United States	3.37%
HSBC Holdings PLC	132.73	Britain	2.11%
Royal Bank of Canada	101.75	Canada	1.62%
Banco Santander SA	74.04	Spain	1.18%
BNP Paribas	58.55	France	0.93%
Lloyds Banking Group PLC	46.28	Britain	0.73%
Intesa Sanpaolo	40.29	Italy	0.64%
Banco Bilbao Vizcaya Argentaria	36.55	Spain	0.58%
Credit Agricole	34.97	France	0.56%
Credit Suisse Group AG	33.37	Switzerland	0.53%
Royal Bank of Scotland Group	31.74	Britain	0.50%
Barclays PLC	28.9	Britain	0.46%
Unicredit SPA	27.88	Italy	0.44%
Société Générale	19.27	France	0.31%
Total	**1 552.45**	**-**	**24.64%**

Banking Sector	**6 300**

The collection of quarterly and annually reports was performed for the years between 2000 and 2018. It is difficult to find reports older than 2000 and the information found in those is more incomplete. The 18-year time frame is adequate in terms of representativeness and is sufficiently large to observe the evolution on VaR disclosures informativeness throughout time. It was only possible to collect a satisfactory and continuous amount of quarterly data for seven of the banks on the sample.

The financial indicators extracted from these reports were the following: Value-at-Risk, Trading Revenue, Total Revenue, Total Assets and Total Equity, being the last three solely for dimensionality purposes.

As stated by Logan and Montgomery (1997) and corroborated by Jorion (2002) findings, it is difficult and quite far-fetched to take considerations on a bank's market risk based on their position on derivatives, as it is not possible to distinguish hedging from speculation activities within this class of assets. This stated, the Notional Amount of Derivatives as a potential alternative proxy for market risk of the bank's trading portfolio was not considered for the purpose of this study.

The obtained numbers were all converted to euros at the respective period (quarter or year) average exchange rate.

There are in total 405 observations, 221 of them constitute quarterly data and 184 yearly data. The final data involves 16 banks, for 7 of which there is quarterly data available and for 15 there is annual data available.

In terms of period coverage, the final data includes 70 quarters and 18 years between 2000 and 2018.

5.3.1. Trading Revenues

Depending on the availability, trading revenues were extracted from quarterly or annual reports, along with VaR disclosures.

As it is possible to observe in table 3, the average trading revenues reported by financial institutions vary widely, from the minimum 397 million euros for

Intesa Sanpaolo, to the maximum of 9 932 million euros for Lloyds Banking Group. The proportion of trading revenues on total revenues is immaterial for some banks, around 1-3%, namely for Wells Fargo, BBVA, Intesa Sanpaolo, Royal Bank of Canada, UniCredit and Banco Santander. For the remaining banks, the trading revenues are more representative, between 4% and 34%, being the maximum registered for Lloyds (33.7%), followed by Société Générale (15%).

Table 3 *- Sample Banks Total versus Trading Revenue (period average)*

	Trading Revenue		Total Revenue	
(in million euros)	**Average**	**Std. Dev.**	**Average**	**% Trading Rev.**
Banco Bilbao Vizcaya Argentaria	433.30	420.66	34 161.65	1.3%
Banco Santander SA	2 200.41	1 380.70	71 454.20	3.1%
Barclays PLC	4 549.69	2 545.13	30 845.33	14.7%
BNP Paribas	5 086.90	1 008.85	65 829.00	7.7%
Credit Agricole	2 950.00	2 764.29	39 260.00	7.5%
Credit Suisse Group AG	2 304.30	3 165.42	53 511.46	4.3%
HSBC Holdings PLC	6 910.60	2 493.65	91 811.54	7.5%
Intesa Sanpaolo	396.62	409.14	25 739.00	1.5%
JPMorgan Chase & Co	7 852.06	5 146.61	90 667.78	8.7%
Lloyds Banking Group PLC	9 931.62	10 705.13	29 491.38	33.7%
Nordea Bank ABP	1 466.54	313.99	15 433.77	9.5%
Royal Bank of Canada	624.23	154.84	44 204.00	1.4%
Royal Bank of Scotland Group	1 725.49	3 649.62	30 234.67	5.7%
Société Générale	6 771.77	2 770.14	45 033.50	15.0%
UniCredit SPA	630.45	1 034.39	39 244.71	1.6%
Wells Fargo & Co	1 007.44	429.65	93 018.00	1.1%

Focusing on the time evolution of trading revenues between 2000 and 2018, presented in figure 2, it is possible to observe that minimum levels were reached during the 2007-08 subprime financial crisis, the most severe since the Great Depression during the 1930s.

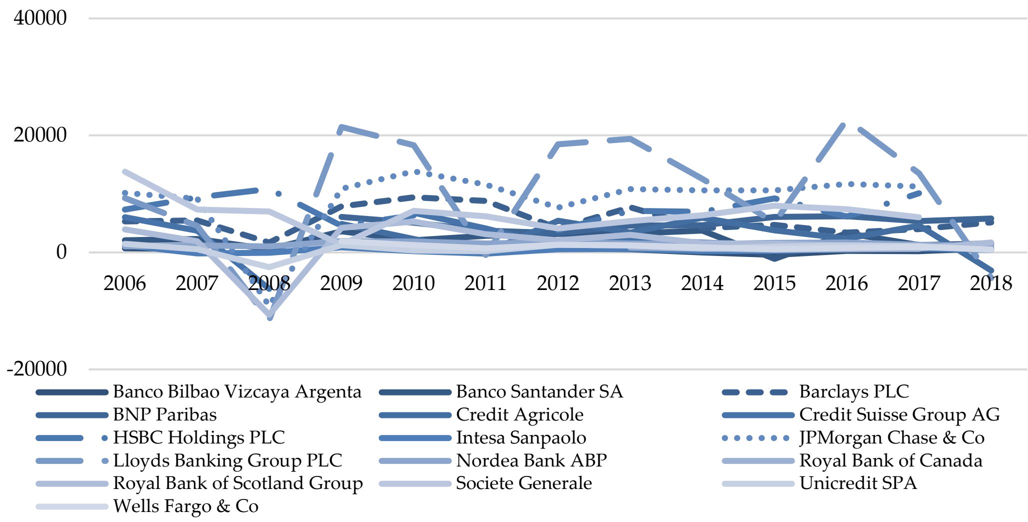

Figure 1 *- Evolution of the Sample Banks' Trading Revenues over the Period (2000-18) in million euros.*

Furthermore, high volatility is observable throughout the period, and confirmed through the significant standard deviations reported on the above table. This shall indicate that unexpected trading revenues will present expressive values.

Table 4 and 5 show that, both yearly and quarterly, the unexpected trading revenues are substantially lower than absolute trading revenues but they are simultaneously different from zero, which allows to reject the hypothesis that the mean of unexpected trading revenues equals zero, consistently with the assumptions made in 5.2.

Table 4 - *Sample Banks' Quarterly VaR-Based Volatility and Unexpected Trading Revenues from 2000 to 2018 (average and standard deviation)*

(in million euros)	VaR		Unexpected Trading Revenue	
	Average	**Standard Deviation**	**Average**	**Standard Deviation**
Banco Santander SA	64	19	121	93
Credit Suisse Group AG	164	69	662	635
JPMorgan Chase & Co	319	208	1 208	1 331
Nordea Bank ABP	202	90	96	79
Royal Bank of Canada	114	73	82	71
Royal Bank of Scotland Group	390	134	313	234
Wells Fargo & Co	64	18	125	128

__Table 5__ - Sample Banks' Annual VaR-Based Volatility and Unexpected Trading Revenues from 2000 to 2018 (average and standard deviation)

	VaR		Unexpected Trading Revenue	
(in million euros)	**Average**	**Standard Deviation**	**Average**	**Standard Deviation**
Banco Bilbao Vizcaya Argentaria	166	29	345	228
Banco Santander SA	158	47	1 643	1 653
Barclays PLC	311	126	1 893	1 802
BNP Paribas	504	443	773	534
Credit Agricole	138	140	3 172	2 049
Credit Suisse Group AG	375	154	3 134	4 063
HSBC Holdings PLC	371	320	2 646	1 507
Intesa Sanpaolo	394	157	444	411
JPMorgan Chase & Co	707	391	3 766	5 941
Lloyds Banking Group PLC	64	81	12 874	9 137
Nordea Bank ABP	477	227	265	225
Royal Bank of Scotland Group	487	427	2 782	4 692
Societe Generale	269	102	2 725	2 216
Unicredit SPA	278	270	998	1 200
Wells Fargo & Co	168	82	375	196

5.3.2. Value-at-Risk

Nowadays, as shown by the work of Perignon et al. (2007) and Perignon and Smith (2008), the industry standard is the one-day VaR, calculated through the historical simulation method and typically the extrapolation to the 10-day VaR, as imposed by the regulator, is calculated simply by scaling using the square root of 10. The review of banks' reports performed for the purpose of this study allowed to confirm such pattern.

Below, there is a summary descriptive table of the collected data (table 6), with the time interval of reports found for each institution, the method used for VaR calculation and its characteristics, meaning, historical time frame, confidence level and horizon.

Table 6 *- Description of the Banks' Reported VaR*

Bank	Reports Available	Method	Time frame	Confidence level	Horizon (days)
Banco Bilbao Vizcaya Argentaria	Yearly: 2006 to 2018	Historical simulation, equally weighted (with smoothings used as suplementary measure)	2	99%	1
Banco Santander SA	Yearly: 2006 to 2018. Quarterly: Q1-09 to Q4-18	Historical simulation; *Statistical adjustments are applied enabling the most recent developments affecting the levels of risk assumed to be incorporated efficiently and on a timely manner.*	2	99%	1
Barclays PLC	Yearly: 2001 to 2018	Historical simulation, equally weighted	2	95%	1
BNP Paribas	Yearly: 2009 to 2018	Monte-Carlo approach, *which not only performs normal or log-normal simulations but also accounts for the non-normality often observed in financial markets as well as correlation : risk factors.*	1	99%	1
Credit Agricole	Yearly: 2009 to 2018	Historical simulation	1	99%	1
Credit Suisse Group AG	Yearly: 2003 to 2018. Quarterly: Q2-03 to Q3-18	Historical (2yr); The model is responsive to changes in market conditions through the use of exponential weighting, which applies a greater weight to more recent events, and the use of expected shortfall equivalent measures to ensure all extreme adverse events are considered in the model.	2	98%	1
HSBC Holdings PLC	Yearly: 2004 to 2017	Historical simulation	2	99%	1
Intesa Sanpaolo	Yearly: 2006 to 2018	Historical simulation	1	99%	1
Lloyds Banking Group PLC	Yearly: 2006 to 2018	Historical simulation	1	95%	1
Nordea Bank ABP	Yearly: 2006 to 2018. Quarterly: Q1-07 to Q4-18	Historical simulation	2	-	
Royal Bank of Scotland Group	Yearly: 2004 to 2018. Quarterly: Q3-09 to Q4-18	Historical simulation, equally weighted	2	99%	1
UniCredit SPA	Yearly: 2006 to 2018	Historical simulation, equally weighted	2	99%	1
Wells Fargo & Co	Yearly: 2009 to 2018. Quarterly: Q1-09 to Q4-18	Historical simulation, equally weighted	1	99%	1
JPMorgan Chase & Co	Yearly: 2000 to 2018. Quarterly: Q1-01 to Q4-18	Historical simulation	1	95%	1
Société Générale	Yearly: 2004 to 2017	Historical simulation	1	99%	1
Royal Bank of Canada	Yearly: 2013 to 2017. Quarterly: Q1-15 to Q3-18	Historical simulation, equally weighted	2	99%	1

All banks, except for BNP Paribas, use historical simulation for VaR reporting purposes, mostly equally weighted (in which the same weight is attributed to all observations) and in fewer cases exponentially decreasing weights to past returns are used, attributing more weight to recent returns (namely, BBVA, Santander and Credit Suisse). The historical basis for calculation fluctuates between one and two years, the confidence level considered is mostly 99% and the horizon 1 day, assuming positions are fixed over a day horizon.

Concerning the data availability, there is more abundancy of observations from 2006, particularly quarterly observations. For some banks it was only possible to find annual reports.

For the purpose of the current work, average VaR was used, as it is more common to be found on banks reports and it also better captures the risk over the period. Using the end of the period VaR, would limit the risk perception to that single day. Furthermore, quarterly average VaR was privileged over yearly average VaR, so whenever quarterly VaR was available and it was possible to match it with a previous quarter trading revenue, it was used.

The hierarchy of reporting frequency choice relates to the fact that annual VaR values are expected to deliver less accuracy as there is a larger time lag to the associated trading revenues.

5.4. Results

Primarily, the model is tested following the steps proposed by Jorion (2002), in order to check how the model with updated data (from 2001 to 2018) compares with the earlier model (data from 1994 to 2000).

The formulated regression is first estimated individually for each bank. Among quarterly observations, seven time-series regressions were tested. Table 7 shows the obtained results under a univariate OLS. It was not tested under the

SUR method because this equation must be balanced, meaning that any information that is only available for some equations will be lost. In this case, much information would be wasted, so this test was excluded.

It is possible to conclude that by applying this new data set, there appears to be some improvement in the results obtained. Credit Suisse and JPMorgan results seem to suggest a statistically significant positive relationship between the volatility measure derived from VaR and unexpected trading revenues. Furthermore, the coefficient of determination (R^2) [16] for these two banks regressions is between 32% and 34%, indicating a good fit of the model.

Table 7 - *Bank Specific Regressions of Quarterly Absolute Unexpected Trading Revenues on VaR-Based Volatility*

$$\left|Rev_{i,t+1} - E[Rev_{i,t+1}]\right| = a_i + b_i\sigma_{i,t} + \varepsilon_{i,t+1}$$

Bank	N. observations	OLS		
		Constant	Slope	R^2
		(t-statistic)		
Banco Santander SA	33	137.29 (2.39)	-0.25 (-0.30)	0.28%
Credit Suisse Group AG	46	-187.12 (-0.91)	5.18*** (4.49)	31.46%
JPMorgan Chase & Co	67	18.50 (0.08)	3.73*** (5.78)	33.97%
Nordea Bank ABP	31	107.03 (2.98)	-0.05 (-0.33)	0.37%
Royal Bank of Canada	9	-13.30 (-0.17)	0.92 (0.87)	9.83%
Royal Bank of Scotland	9	364.99 (1.36)	-0.13 (-0.20)	0.58%
Wells Fargo & Co	26	62.31 (0.64)	0.97 (0.66)	1.8%

$Rev_{i,t+1}$ denotes the quarterly trading revenue at time t+1 for bank i; $E[Rev_{i,t+1}]$ denotes the moving average of the previous four quarters trading revenues, for bank i; $\left|Rev_{i,t+1} - E[Rev_{i,t+1}]\right|$ is the explained variable and denotes the unexpected absolute trading revenue for bank i at time t; $\sigma_{i,t}$ is the explanatory variable and denotes the volatility measure based on VaR at time t for bank i.

* denotes p-value ≤ 0.05 ** denotes p-value ≤ 0.01 *** denotes p-value ≤ 0.001.

[16] Assesses the fit of the equation, comparing it to one with no explanatory variables that uses the mean of the explained variable as sole predictor.

Moving on to the full estimation of the equation, presented on table 8, a pooled sample OLS was regressed, presenting a VaR based volatility coefficient of 3.37, significant for a p-value below 0.001. The R^2 is 36.96%, which evidences a good model fit.

***Table 8** - Pooled Regressions of Quarterly Absolute Unexpected Trading Revenues on VaR-Based Volatility*

$$\left|Rev_{i,t+1} - E[Rev_{i,t+1}]\right| = a_i + b_i\sigma_{i,t} + \varepsilon_{i,t+1}$$

	N. observations	**Constant** (t-statistic)	**Slope**	**R^2**
Pooled Sample OLS	221	-92.50 (-1.21)	3.37*** (11.32)	36.96%
Heteroskedasticity-robust	221	-92.50 (-0.86)	3.23*** (4.79)	36.65%

$Rev_{i,t+1}$ denotes the quarterly trading revenue at time t+1 for bank i; $E[Rev_{i,t+1}]$ denotes the moving average of the previous four quarterly trading revenues, for bank i; $\left|Rev_{i,t+1} - E[Rev_{i,t+1}]\right|$ is the explained variable and denotes the unexpected absolute trading revenue for bank i at time t; $\sigma_{i,t}$ is the explanatory variable and denotes the volatility measure based on VaR at time t for bank i.

* denotes p-value ≤ 0.05 ** denotes p-value ≤ 0.01 *** denotes p-value ≤ 0.001.

Some evidences of heteroscedasticity are found in figure 3 with an increasing dispersion of unexpected trading revenues as volatility based on VaR increases. There is also evidence in figure 2, particularly on the 2008 trading revenues drop.

Considering this, a heteroscedasticity-robust t statistic was also computed (see table 8), and similar results to the Pooled Sample OLS were obtained.

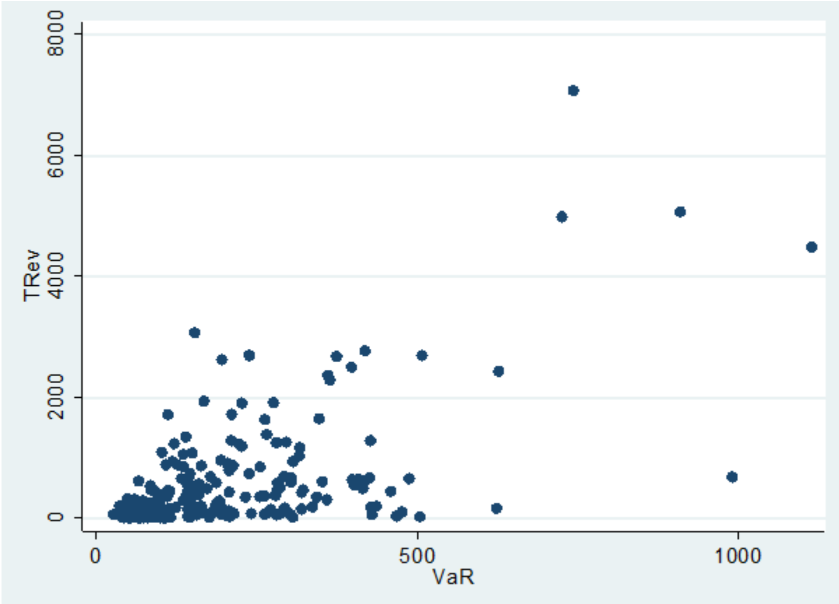

Figure 2 - Quarterly VaR-Based Volatility in Absolute Unexpected Trading Revenues

To control for potential endogeneity issues, considering the sample combines cross section with time series, an iteration of the model with fixed effects (FE) is performed. The FE technique adds a group of dummy variables to control for the existence of omitted variables that are assumed constant across group members but may vary across groups.

The above was performed for both banks and dates and the results are presented on table 9.

Table 9 - Pooled Regressions of Quarterly Absolute Unexpected Trading Revenues on VaR-Based Volatility - Fixed Effects

$$\left|Rev_{i,t+1} - E[Rev_{i,t+1}]\right| = a_i + b_i\sigma_{i,t} + \varepsilon_{i,t+1}$$

	N. observations	Constant (t-statistic)	Slope (t-statistic)	R^2
FE applied to banks	221	-96.15 (-1,21)	3.37*** (9,32)	47.85%
FE applied to dates	221	-980 (-1.65)	2.17*** (5.58)	73.83%

$Rev_{i,t+1}$ denotes the quarterly trading revenue at time t+1 for bank i; $E[Rev_{i,t+1}]$ denotes the moving average of the previous four quarterly trading revenues, for bank i; $\left|Rev_{i,t+1} - E[Rev_{i,t+1}]\right|$ is the explained variable and denotes the unexpected absolute trading revenue for bank i at time t; $\sigma_{i,t}$ is the explanatory variable and denotes the volatility measure based on VaR at time t for bank i.

* denotes p-value ≤ 0.05 ** denotes p-value ≤ 0.01 *** denotes p-value ≤ 0.001.

It is possible to understand that in both cases the VaR based volatility is significant for a p-value of 0.001 and the R^2 is 48% and 74%, for the FE applied to banks and dates respectively, showing a very satisfactory model fitting when controlling for endogeneity. The VaR based coefficient is similar to the obtained with the OLS model, of 3.37 and 2.17, when controlling for banks and dates, respectively.

Moving to the annual database, the same procedures were performed and worse results are expected due to the already mentioned loss of accuracy when increasing the time frame between the reported VaR and the reported trading revenues.

Primary, the formulated regression is estimated individually for each bank. Among yearly observations, sixteen time series regressions were tested. Table 10 shows the obtained results under a univariate OLS. It was not tested under the SUR method because this equation must be balanced, meaning that any information that is only available for some equations will be lost. In this case, much information would be unexploited, so this test was excluded.

It is possible to conclude that the results of Credit Suisse Group AG and UniCredit SPA seem to suggest a statistically significant relationship between the volatility measure derived from VaR and unexpected trading revenues. Furthermore, the coefficient determination (R^2) for these two banks regressions is of 74% and 57%, indicating a good fit of the model. The remaining banks isolated, did not present significant coefficients for the VaR based volatility.

Table 10 - Bank Specific Regressions of Yearly Absolute Unexpected Trading Revenues on VaR-Based Volatility

$$\left|Rev_{i,t+1} - E[Rev_{i,t+1}]\right| = a_i + b_i\sigma_{i,t} + \varepsilon_{i,t+1}$$

Bank	N. observations	OLS Constant (t-statistic)	OLS Slope (t-statistic)	R^2
Banco Bilbao Vizcaya Argentaria	8	749.49 (1.48)	-2.44 (-0.81)	9.85%
Banco Santander SA	12	3277.81 (1.86)	-10.34 (-0.97)	8.54%
Barclays PLC	17	-84.31 (0.940)	6.35 (1.91)	19.64%
BNP Paribas	9	659.11 (2.25)	0.23 (0.50)	3.50%
Credit Agricole	9	3427.33 (3.27)	-1.85 (-0.34)	1.59%
Credit Suisse Group AG	12	-5378.63 (-3.16)	22.70*** (5.37)	74.27%
HSBC Holdings PLC	12	2092.71 (3.08)	1.49 (1.06)	10.09%
Intesa Sanpaolo	12	911.64 (2.93)	-1.19 (-1.61)	20.58%
JPMorgan Chase & Co	17	-432.98 (-0,15)	5.94 (1.64)	15.24%
Lloyds Banking Group PLC	12	14420.94 (4.11)	-24.19 (-0.69)	4.58%
Nordea Bank ABP	12	319.02 (1.96)	-0.11 (-0.36)	1.31%
Royal Bank of Canada	4	59.57 (0.33)	0.41 (0.48)	10.48%
Royal Bank of Scotland Group	14	2448.18 (1.25)	0.69 (0.22)	0.39%
Societe Generale	13	1536.92 (0.84)	4.42 (0.69)	4.15%
Unicredit SPA	12	62.35 (0.18)	3.37** (3.67)	57.35%
Wells Fargo & Co	9	154.43 (1.10)	1.30 (1.73)	29.93%

$Rev_{i,t+1}$ denotes the quarterly trading revenue at time t+1 for bank i; $E[Rev_{i,t+1}]$ denotes the moving average of the previous four quarterly trading revenues, for bank i; $\left|Rev_{i,t+1} - E[Rev_{i,t+1}]\right|$ is the explained variable and denotes the unexpected absolute trading revenue for bank i at time t; $\sigma_{i,t}$ is the explanatory variable and denotes the volatility measure based on VaR at time t for bank i.

* denotes p-value ≤ 0.05 ** denotes p-value ≤ 0.01 *** denotes p-value ≤ 0.001.

Moving on to the full estimation of the equation, a pooled sample OLS was regressed, presenting a non-significant VaR based coefficient.

Table 11 - Pooled Regressions of Yearly Absolute Unexpected Trading Revenues on VaR-Based Volatility

$$\left|Rev_{i,t+1} - E[Rev_{i,t+1}]\right| = a_i + b_i\sigma_{i,t} + \varepsilon_{i,t+1}$$

	N. observations	Constant	Slope	R^2
		(t-statistic)		
Pooled Sample OLS	184	2533.67 (4.91)	0.17 (0.15)	0.01%
Heteroskedasticity-robust	184	2533.67 (3.83)	0.17 (0.11)	0.01%

$Rev_{i,t+1}$ denotes the quarterly trading revenue at time t+1 for bank i; $E[Rev_{i,t+1}]$ denotes the moving average of the previous four quarterly trading revenues, for bank i; $\left|Rev_{i,t+1} - E[Rev_{i,t+1}]\right|$ is the explained variable and denotes the unexpected absolute trading revenue for bank i at time t; $\sigma_{i,t}$ is the explanatory variable and denotes the volatility measure based on VaR at time t for bank i.

* denotes p-value ≤ 0.05 ** denotes p-value ≤ 0.01 *** denotes p-value ≤ 0.001.

Some extra evidences of heteroscedasticity are found in figure 4, compared with the quarterly basis, with an increasing dispersion of unexpected trading revenues as volatility based on VaR increases. There is also evidence in figure 2, particularly on the 2008 trading revenues drop.

Considering this, a heteroscedasticity-robust t statistic was also computed, and similar results to the Pooled Sample OLS were obtained.

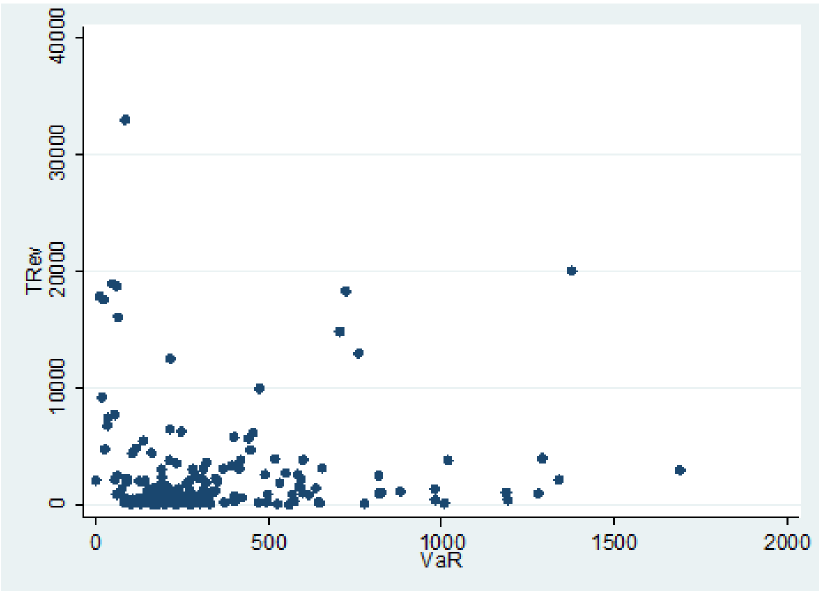

Figure 3 - Yearly VaR-Based Volatility in Absolute Unexpected Trading Revenues

To control for potential endogeneity issues, considering the sample combines cross section with time series, an iteration of the model with fixed effects (FE) is performed (see table 12).

The above was performed for both banks and dates (time-series and cross sectionally) and the results are presented below.

Table 12 - *Pooled Regressions of Yearly Absolute Unexpected Trading Revenues on VaR-Based Volatility - Fixed Effects*

$$|Rev_{i,t+1} - E[Rev_{i,t+1}]| = a_i + b_i\sigma_{i,t} + \varepsilon_{i,t+1}$$

	N. observations	**Constant** (t-statistic)	**Slope**	**R^2**
FE applied to banks	184	-87.72 (-0.07)	2.61* (2.34)	44.36%
FE applied to dates	184	2529 (-1.65)	-1.25 (-1.05)	23.36%

$Rev_{i,t+1}$ denotes the quarterly trading revenue at time t+1 for bank i; $E[Rev_{i,t+1}]$ denotes the moving average of the previous four quarterly trading revenues, for bank i; $|Rev_{i,t+1} - E[Rev_{i,t+1}]|$ is the explained variable and denotes the unexpected absolute trading revenue for bank i at time t; $\sigma_{i,t}$ is the explanatory variable and denotes the volatility measure based on VaR at time t for bank i.

* denotes p-value ≤ 0.05 ** denotes p-value ≤ 0.01 *** denotes p-value ≤ 0.001.

It is possible to understand that the VaR based volatility coefficient is significant for a p-value of 0.05 in the FE applied to banks, with an R^2 of 44%, showing a satisfactory model fitting when controlling for endogeneity. However, the VaR based volatility when using FE applied to dates does not appear as significant.

The overall performance of the same models, when applied to yearly data demonstrates much poorer performance then when applied to quarterly data, showing that the closer the VaR reporting is to the following trading revenues report, the better the explanatory power of the first over the second.

Summarizing, it is possible to infer that the VaR based volatility improved its performance over the years, considering the results obtained in this study. When compared to Jorion (2002) paper, the current results produce better R^2, meaning a relative increase in the model's fit, and larger coefficients at a lower p-value level, which increases certainty on the significance of VaR when explaining unexpected trading revenues.

Conclusion

VaR is an evident keystone indicator in the market risk management sphere, generally accepted by stakeholders as a reliable measure of an institution's maximum potential losses related to market risk. Therefore, the core aim of this study is to understand how informative the VaR publicly disclosed by financial institutions under their mandatory reporting is.

Until now, the only precedent for this study was developed by Jorion (2002), who at the time concluded that *the informativeness of VaR disclosures in firms' financial reports was of considerable interest*. He added that, over time, as the methodologies become more consistent and the reports available more abundant, one *would expect the VaR-based volatility to become increasingly accurate indicator of the variability of banks' future trading revenues*.

The current study, by updating and enlarging the sample for the period of 2000-2018 and 16 large banks, was able to successfully confirm the increase in VaR informativeness. This is supported on consecutive methodological evolutions over the past years combined with the emergence of backtesting techniques, which were gradually embedded in the banking industry.

The VaR publicly disclosed by financial institutions in their quarterly reports demonstrated very satisfactory informativeness on the subsequently reported trading revenues. As expected from the beginning of this study, when running and testing the same regression using a sample of annual reported data, the results were not as satisfactory.

From the results obtained, VaR reinforces its position as a valuable instrument for stakeholders when evaluating the market risk profile of financial institutions.

An evident limitation of this study was the unbalanced data both in time series and cross-sectionally, which creates boundaries to the scope of tests being applied. Simultaneously, correcting this unbalancing would imply losing too much information. It would be of great interest to perform similar work having a considerably large sample of balanced data available.

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